The children in Ms. Hunter’s Head Start classroom are fascinated whenever Ms. Winter, the assistant teacher, brings Barry, a large bat puppet, out of his cave to help them solve math problems. Sitting on Ms. Winter’s knee, Barry tells the children that he needs their help to fix a hole in his cave (a decorated cardboard and papier-mâché creation in the corner of the room). The children inspect the cave and find the hole. Ms. Hunter suggests that perhaps rocks will work to make the repair.

Ms. Hunter shows the children a large tray of rocks fairly equal in size and two different-size empty boxes.

Barry asks the children questions and reviews vocabulary they might need to talk about the problem: “How will I use the rocks to repair the cave’s hole?,” “How many rocks do you think I need?” Ms. Winter writes the numbers the children predict on a piece of chart paper. “Which is the larger box? Which box is smaller?” Ms. Hunter shows the children how one box fits into the other, and she asks the children to predict which box they think will hold more rocks and which box will hold fewer.

The children take turns placing rocks in the boxes until they observe that both boxes are full. Then the chil-
The problem-solving and critical thinking skills the children use to help Barry fix the cave are an important part of early mathematics development. Young children have plenty of curiosity about the world around them, and they like to figure things out about it (Clements & Sarama 2009, 2012). Mathematics includes important tools for doing just that. Between the ages of 3 and 6, children begin to advance in their abilities to use analogical reasoning (making connections between prior knowledge and newly presented information), generalize, and plan ahead (Ginsburg, Ertle, & Presser 2013). Using these important thinking skills in the context of mathematics is as critical to children’s academic readiness as acquiring specific math content such as number recognition (Clements & Sarama 2009). The joint position statement of NAEYC and the National Council of Teachers of Mathematics (NCTM) points out that high-quality mathematics education before first grade should “use curriculum and teaching practices that strengthen children’s problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas” (NAEYC & NCTM [2002] 2010, 3).

**NCTM content standards and process standards**

Prekindergarten mathematics education is included in NCTM’s Principles and Standards for School Mathematics (2000), providing a scope and sequence for mathematics curriculum through grade 12. NCTM identifies five content standards: number and operations, algebra, geometry, measurement, and data analysis and probability. These content areas receive different emphases across grades. In prekindergarten, the core of mathematics instruction rests in the number and geometry standards, and “each of the other standards contributes to and is learned in conjunction with these two standards” (76). NCTM also identifies process standards, including problem solving, making connections, using reasoning, representing mathematical ideas and operations, and communicating (see “Mathematical Content and Process Areas”).

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Teachers can then intentionally apply the NCTM process standards across the mathematics content areas. It is important for teachers to include problem solving in each mathematics content area (see “Problem-Solving Ideas Across NCTM Mathematical Content Areas”).

Although many early childhood educators recognize the importance of including well-planned mathematics instruction throughout the day and relish the opportunity to help children engage in mathematics problem solving and critical thinking, finding instructional resources can be difficult. Mathematics often receives less emphasis or may be omitted altogether from commonly used preschool curricula (Preschool Curriculum Evaluation Research Consortium 2008). When mathematics instruction is included in these curricula, the focus is frequently limited to teaching just numbers and operations (number recognition and counting) and geometry (shapes) (Butera et al. 2011). Yet school curricula are beginning to focus more often on science, technology, engineering, and mathematics (STEM) from the earliest years of children’s schooling (Clements & Sarama 2009; Moomaw & Davis 2010). Under such circumstances, the infusion of NCTM mathematics process standards into preschool curricula is critical and will connect children’s learning experiences from preschool to learning in the early primary grades.

### Problem-Solving Ideas Across NCTM Mathematical Content Areas

<table>
<thead>
<tr>
<th>Math content area</th>
<th>Content area focus</th>
<th>Problem-solving ideas in math</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and operations</td>
<td>Counting, comparing, adding to and taking away, grouping</td>
<td>■ Comparing numbers of objects (e.g., Who has more rocks?)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Understanding addition or subtraction of objects</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Representing a number in different ways (e.g., writing the numeral 5 and having five rocks)</td>
</tr>
<tr>
<td>Measurement</td>
<td>Figuring how much, using tools, comparing, identifying attributes of objects (length, height, weight)</td>
<td>■ Comparing length, area, capacity, weight, time (e.g., Who is taller? Is the car heavier than the bike?)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Exploring the use of different measurement tools</td>
</tr>
<tr>
<td>Geometry</td>
<td>Naming shapes, recognizing transformations in size, recognizing symmetry, visualizing, using spatial reasoning</td>
<td>■ Building shapes with connecting blocks and tangrams</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Using parquetry pattern cards to match shapes, figure out which cards go where</td>
</tr>
<tr>
<td>Algebra</td>
<td>Understanding patterns, predicting</td>
<td>■ Putting pegs in a Peg-Board using a pre-set or new pattern</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Stringing beads to match a model or recurring order (e.g., Can you make one like this?)</td>
</tr>
<tr>
<td>Data analysis and probability</td>
<td>Classifying, organizing, representing, using information to ask and answer questions</td>
<td>■ Charting the number of people in children’s families (e.g., Are there more brothers than sisters?)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>■ Tallying the different types of weather through the year</td>
</tr>
</tbody>
</table>

& Boynton 2003; Copley 2010) to describe how preschool teachers can embed problem solving and critical thinking across a typical day’s activities as well as in intentional daily mathematics lessons. The aim is to encourage children to think critically about solutions rather than focus on one “right” answer. Teachers are purposeful in teaching a problem-solving process as part of their CSS instruction, including in mathematics, and children learn to use problem-solving steps across many curriculum activities.

The use of the integrated CSS curriculum significantly improved child outcomes in our efficacy study of it, particularly in assessments of mathematical reasoning (Odom et al. 2003). Using data collected from 783 children in five states (California, Kansas, Indiana, West Virginia, Maryland), the results demonstrated significantly improved mathematics outcomes for children using the CSS curriculum compared to children using traditional mathematics curricula (Odom et al. 2010).

We are encouraged by these results and have undertaken a revision of the CSS curriculum to strengthen its features that are especially important in supporting children’s opportunities for problem solving and critical thinking. On the following pages, we present some of the lessons about children’s use of critical thinking and problem solving in the context of mathematics that we learned while using the CSS curriculum. (These ideas are summarized in “CSS Ideas for Facilitating Young Children’s Mathematical Problem Solving and Critical Thinking,” on p. 74.)

Providing instructional support
Preschool children universally learn by doing, interacting with materials, and drawing conclusions or testing hypotheses about things that are new to them. Teachers can scaffold children’s hands-on learning by preparing and teaching using a structure for problem solving. With the CSS curriculum, teachers found success in promoting children’s critical thinking by using an intentional problem-solving process with activities. This included supporting children by scaffolding instruction in the following four steps.

Scaffolding Instruction for Children to Solve Problems
1. Reflect and ask
2. Plan and predict
3. Act and observe
4. Report and reflect

(French, Conezio, & Boynton 2003)

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For example, in one activity children learned the different likes and dislikes of each child in the classroom through discussion in which they analyzed data. Ms. Hunter presented three fruits during snack time—mango, kiwi, and banana—and encouraged the children to reflect on and ask questions about what they observed or already knew about each of the fruits and ask questions (Reflect and ask). Then the teacher asked the children to plan what they would need to do to taste each fruit (e.g., peel, cut, remove seeds) and to predict what each fruit would taste like and which would be the most popular in the class. Ms. Hunter charted their answers in a simple bar graph (Plan and predict). The children tasted all the fruits and talked with each other about the flavors and textures (Act and observe). Ms. Hunter then charted their favorites and facilitated a class discussion about whether their predictions were accurate and what they had learned (Report and reflect).

To teach reasoning skills, CSS teachers showed the children how to use an explicit sequence of five problem-solving steps so they might begin to understand what a problem is and how to use logical reasoning to attempt to solve it:

1. What is the problem?
2. Think of some solutions that might work.
3. Talk about solutions and pick the best one.
4. Try it!
5. How did it work?

For example, as part of an apple theme, Ms. Winter presented children with the problem of how to repeat a pattern (green, yellow, red) of apples. She asked the children how they could continue the pattern (e.g., “Should we guess?,” “Should we count?,” “Should we draw a picture?”). Together, they decided that grouping a number of apples might be a good strategy. Upon trying it, they discovered that they could clearly see how the pattern of a green, a yellow, and a red apple repeats. Reflecting on the process, the children agreed that they now knew how to continue a pattern and work together to do so. During center work, children created their own patterns using different materials such as crayons, playdough, and manipulatives.

It is important to include NCTM's mathematics content areas and process standards throughout the children's daily activities.

Integrating mathematics problem solving and critical thinking across the CSS curriculum

It is important to include NCTM’s mathematics content areas and process standards throughout the children’s daily activities, as well as set aside a time for mathematics instruction (Klibanoff et al. 2006). In the mathematics lesson with Barry, the children used numbers and operations to count the number of rocks filling the boxes, and then they used measurement when they compared the number of rocks in each box. They used geometry when they compared box sizes. In addition, Barry the Bat and the rocks appeared throughout the day in Ms. Hunter’s classroom, and several times she referenced the ideas about mathematics that the children discussed in the cave repair mathematics lesson. The children gathered and counted rocks during outside play.

CSS Ideas for Facilitating Young Children’s Mathematical Problem Solving and Critical Thinking

Provide Instructional Support
- Provide children with a structure for problem solving (see “Scaffolding Instruction for Children to Solve Problems,” p. 73)
- Teach children an explicit series of problem-solving steps (see the five steps above)

Integrate Mathematics Problem Solving and Critical Thinking Across the Day
- Plan to both directly teach and integrate mathematics content areas and process standards into your day (see “Mathematical Content and Process Areas,” p. 71)

Consider Individual Differences
- Adapt activities to accommodate all children’s learning
- Provide a framework for thinking about adaptations and making curriculum modifications, such as environmental support, material adaptations, simplification of the activity, use of child preferences, special equipment use, invisible support, adult support, and peer support (Sandall et al. 2008).
Later in the day Ms. Winter read the picture book *Rockheads*, by Harriet Ziefert. The book uses a poem, paired with cartoon figures with heads made of rocks, to count to 12. Before reading the story, the teachers gave each child a small and a large rock, similar to those seen in the story, and asked which one was heavier and which was lighter. In the story, the rockheads gather one by one, providing an opportunity for the children to stop and count the rocks and compare their sizes. Ms. Winter also promoted critical thinking about number patterns (“I wonder how many rockheads will be on the next page?”).

After reading the book, the children made their own rockheads, using the two rocks they had been given, glue, and markers. Ms. Winter wondered aloud how many rockheads were in the class, and the activity ended with the class counting them together. They encouraged the children to experiment with the rocks using balancing scales.

**Providing for individual differences**

Early childhood educators are well aware that young children develop skills in mathematics in their own way and at their own pace, just as they do when developing early literacy skills (NAEYC & NCTM [2002] 2010). Curricula must provide ways for teachers to adapt activities easily to accommodate these differences. Sandall and colleagues (2008) describe strategies for making curriculum modifications in *Building Blocks for Teaching Preschoolers With Special Needs*. Curriculum modifications—such as environmental support, material adaptations, simplification of the activity, use of child preferences, special equipment use, invisible supports (e.g., adjusting the schedule to accommodate all learners), adult support, and peer support—provide a framework for thinking about how to help children with different needs learn mathematics.

**Curriculum modifications, adult support, and peer support provide a framework for thinking about how to help children with different needs learn mathematics.**

In the cave repair lesson, Ms. Hunter and Ms. Winter worked together in the classroom to provide adult support to children who needed it in order to engage in the lesson and respond appropriately. They talked often about the specific progress children were making and noted which children still struggled with one-to-one correspondence and how they could promote their understanding through intentional center work or individual support.
Ms. Winter often used Barry to help keep children engaged. She and the bat puppet moved around the classroom, with Barry talking to Ms. Hunter and the children as they did so. The teachers also used the adaptation of simplifying the activity when the class made their own rockheads: the teachers glued some of the rocks together before the activity so that some children had fewer steps to complete. They also used peer support by asking children who had finished their rockheads to help children who had not.

**Conclusion**

Teaching preschool children to problem solve and engage in critical thinking in the context of mathematics instruction requires a series of thoughtful and informed decisions. Despite its importance, mathematical professional development often receives less priority than other areas, such as literacy. This is a concern because some early educators may lack a deep understanding of mathematics, may avoid instruction due to their own insecurities in math, or may question whether mathematical instruction is appropriate for young children (Ginsburg et al. 2006). Nevertheless, providing informal opportunities for problem solving in the preschool classroom can be accomplished. Teacher modeling of problem solving, combined with guidance during children’s problem solving, is critical to promoting children’s learning.

The Children’s School Success curriculum study demonstrated that a problem-solving approach to mathematics in an integrated curriculum is effective. Mathematics is an area of interest to young children, and teachers can build on problem solving throughout the curriculum to influence the developmental processes that are part of mathematical understanding.

**References**


